

Aerial Shell Drift Effects

K.L. and B.J. Kosanke

PyroLabs, Inc., 1775 Blair Road, Whitewater, CO 81527, USA

ABSTRACT

A prime consideration in determining separation distance requirements for aerial fireworks displays is where fallout of dangerous debris is likely to occur. Certainly the most dangerous single piece of fallout is a dud aerial shell. Thus it is important to have knowledge of where duds may fall during typical displays. This would be a relatively simple situation if aerial shells were ballistically stable, and they precisely followed the path determined by mortar orientation, shell muzzle velocity, and atmospheric conditions. Unfortunately, however, aerial shells tend to drift from their ideal (predicted) path, and that drift is greater than most realize. In order to determine where dud shells fall, a large number of aerial shells, both spherical and cylindrical, were fired into the air after having been rendered incapable of bursting at altitude. Most firings were from mortars that were positioned vertically and under calm wind conditions; however, some firings were from angled mortars. For spherical aerial shells, 7.6 cm to 25.4 cm (3 in. to 10 in.) it was found that, on average, duds fall 3.8 m per cm (32 ft per in.) of shell size, from the point ballistically predicted. Further the data suggests that drifts as great as 12 m per cm (100 ft per in.) of shell size may occur nearly 1 percent of the time. For cylindrical shells, 7.6 cm to 15.2 cm (3 in. to 6 in.) it was found that, on average, duds fall 2.4 m per cm (20 ft per in.) of shell size, from the point ballistically predicted. Finally, a large number of 10.2-cm (4-in.) cylindrical shells were fired in order to determine the effect of shell weight, shell length, and lift powder weight on drift distance.

Introduction

When aerial shells are fired from a mortar, fairly accurate predictions can be made about their ideal (average or typical) trajectories, providing the necessary input information is available. The type of information needed includes the shell's

shape, weight, amount and type of lift, mortar tilt angle and azimuth, wind speed and direction. These ballistic predictions could be based on empirical data, but more often they are based on mathematical calculations.^{1,2,3} The accuracy of the predictions generally improve with more and better input information. However, at present, the exact trajectory for an individual shell is not predictable. This is because, for each individual shell being fired, other needed input information is unknown or unknowable, and the mathematical models presently available lack the degree of sophistication to use the information even if it were known. For the purpose of this paper, the difference between the ballistically predicted path of an aerial shell and its actual path will be termed drift distance.

Knowledge of aerial shell drift distance is important in establishing appropriate spectator separation distances for fireworks displays. For example, if it were possible to align all of one's mortars so as to cause all shells to be propelled toward one specific fallout point, then it would be easy to avoid injuries from duds falling into the crowd. However, because individual shell drifts cannot be predicted, it is not possible to aim each mortar to compensate for drift. Thus, dud shells occurring during a show will be scattered about the fallout area. In establishing appropriate spectator separation distances, it is important to know how widely those duds are likely to be scattered.

The reason aerial shells drift or wander from their ideal trajectories is not completely known, at least by the authors. However, the cause of drift is of less concern than is its magnitude. The reason aerial shell drift has more than mere academic interest, is that drift effects are considerably greater than many in the fireworks display business realize. This, in turn, means that appropriate spectator separation distances are greater than many realize. This paper summarizes information presented earlier by the authors^{4,5} and others,^{6,7} and

presents the results of new work by the authors and others⁸.

Background Information

Magnus Effect

While there may be many causes for aerial shell drift, it may be useful to discuss one cause as an example of how drift forces arise. Aerial shells generally tumble through the air after they are fired from a mortar. This tumbling (spinning) produces an effect analogous to that when a baseball pitcher throws a curve ball. The tumbling shell follows a trajectory which curves (i.e., drifts) away from that predicted based solely on mortar tilt and wind effects. This tumbling or curve-ball effect is technically known as the Magnus effect.

The magnitude of the drift derived from the Magnus effect depends on the rate of spin of the shell and its velocity through the air. To better understand why this is the case, consider Figure 1(A). Here a rotating aerial shell is depicted with air flowing past it. (From a physics standpoint this is the same as if the shell was moving through still air, but a stationary shell and the forces acting on it are easier to visualize and draw.) As the shell rotates, a thin layer of air, called the boundary layer, rotates along with the shell. When this air motion is combined with that moving past the shell, the resulting air velocity will not be the same on both sides of the shell. In effect, the two air motions add on the left side of the shell producing a higher overall velocity, and they subtract on the right side producing a lower velocity. Bernoulli's Principal states that the pressure in a mov-

ing column of fluid is inversely proportional to its velocity. Although not completely applicable in this case, it suggests that the pressure acting on the left side of the shell (P_1) will be less than that on the right side (P_2), see Figure 1(B). This pressure differential produces a net force (F) acting on the shell toward the left. This is the Magnus force, and it acts to push a rotating shell off course. The magnitude of the drift depends on the magnitude of the force and the length of time the force is applied. This depends on the combination of the shell's velocity through the air, its rate of spin, and the duration of the flight of the shell (with greater flight times resulting in greater drifts).

Mortar Characteristics

The velocity of a shell and its time of flight depend (to some extent) on the characteristics of the mortar being used. For this reason, Table 1 gives characteristics of the mortars used in these tests and indicates on which shell drift tests they were used.

Air Density

Another factor possibly influencing drift distance is air density, which depends on the elevation above sea level at which shells are fired. As air density decreases, so does the magnitude of forces acting on the shell; drift forces will be less, but so will aerodynamic drag (which means the flight time of shells will be greater). The combined effect of increased elevation is to have a smaller drift force acting for a longer time. At the time of writing this paper, the authors have not evaluated the precise net effect of changes in elevation; however, it is felt that any elevation de-

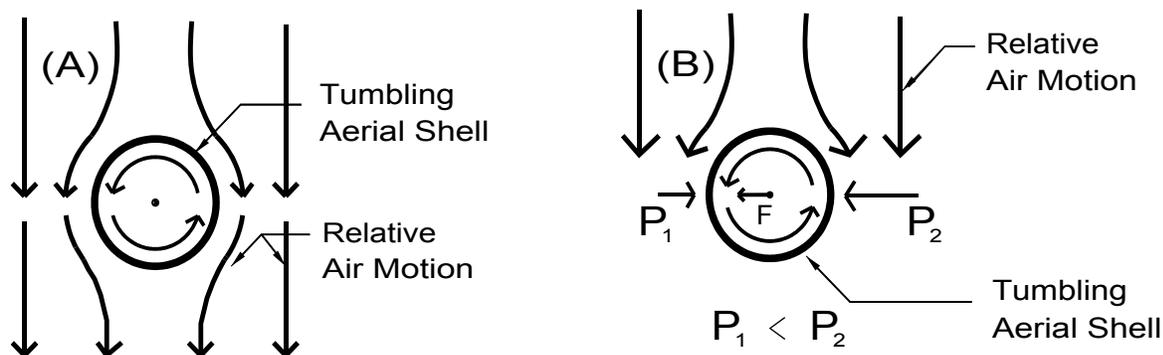


Figure 1. Rotating spherical aerial shell with air flowing past it, and the force produced as a result of pressure differential ($P_2 - P_1$).

Table 1. Characteristics of Mortars Used in Aerial Shell Drift Tests.

Shell Size		Mortar Length ^(a)		Mortar I.D.		Mortar Material ^(b)	Type of Shell Fired Used
cm	(in.)	cm	(in.)	cm	(in.)		
7.6	(3)	57.2	(22.5)	7.3	(2.89)	HDPE	All
10.1	(4)	56.9	(22.4)	9.9	(3.88)	HDPE	Spherical
10.1	(4)	61.0	(24.0)	10.4	(4.10)	HDPE ^(c)	Cylindrical
12.7	(5)	68.6	(27.0)	12.5	(4.92)	HDPE	All
15.2	(6)	68.6	(27.0)	15.0	(5.91)	HDPE	Spherical
15.2	(6)	75.7	(29.8)	15.5	(6.10)	Steel	Both
20.3	(8)	98.6	(38.8)	20.3	(8.00)	Steel	Spherical
25.4	(10)	118.9	(46.8)	25.6	(10.06)	Steel	Spherical

(a) Mortar length is measured from the top of the mortar plug to the mouth of the mortar.

(b) HDPE = High Density Polyethylene.

(c) Mighty-Mite mortar with a slightly tapering ID. Reported ID is an average.

pendence is small. (Except as noted, the tests reported here were conducted in Whitewater, CO at approximately 4600 ft above sea level.)

The effect of temperature and pressure variations, which affect air density, is also expected to be small.

Absolute Drift Predictions

The speed of an aerial shell as it leaves the mortar and then travels through the air is roughly predictable based on calculations using typical shell parameters,^{1,2} or it can be measured.^{9,10} However, as suggested in the introduction, the magnitude and orientation of a shell's drift are not absolutely predictable. In part, this is because no one has developed an adequate mathematical model. However, more importantly, when a shell is fired, one does not have details of the shell's exact position in the mortar, the shell's internal mass distribution, the smoothness and symmetry of the shell's surface and the mortar's interior, etc., all of which would be needed to perform a drift calculation (assuming an appropriate mathematical model existed). For this reason, it may never be possible to calculate drift distance for an individual shell. As an alternative it is possible to measure typical aerial shell drifts; then to use this information in a general way to predict the average drifts of shells to be fired. However, it must be recognized that those predictions will only be accurate in a statistical sense. For example, it might be possible to state for a given type of shell, that 5% of the time it will drift between 30 and 60

meters in a direction between north and east. Similarly, the likelihood for other drifts could be stated. However, for any particular shell, it is not possible to predict the precise magnitude or direction of its drift. For this reason, drift distances reported in this paper can only be stated in a statistical or probabilistic sense.

Experimental Method

Shell Firings

Except in a few cases, the only mortar orientation used in this study was vertical. Also, tests were generally performed under calm surface wind conditions, i.e., winds measured at 6.1 meters (20 ft) above ground were less than 3.2 km/h (2 mph). Both spherical and cylindrical shells were tested. The spherical aerial shells were commercially produced and ranged in size from 7.6 cm (3 in.) to 25.4 cm (10 in.). Before being fired, the shells were altered so as not to burst during their flight (i.e., they were made into duds). (In most cases this was accomplished by injecting water into each shell's time fuse.) Also, a variety of shells from different manufacturers were used, so that the results would tend to be independent of peculiarities of any one manufacturer. (The brands or manufacturers used were Onda, Yung Feng, Horse, Temple of Heaven, and Flying Dragon.) The shells were fired using an electric match to replace the quick match shell leader installed by the manufacturer. All the cylindrical shells were

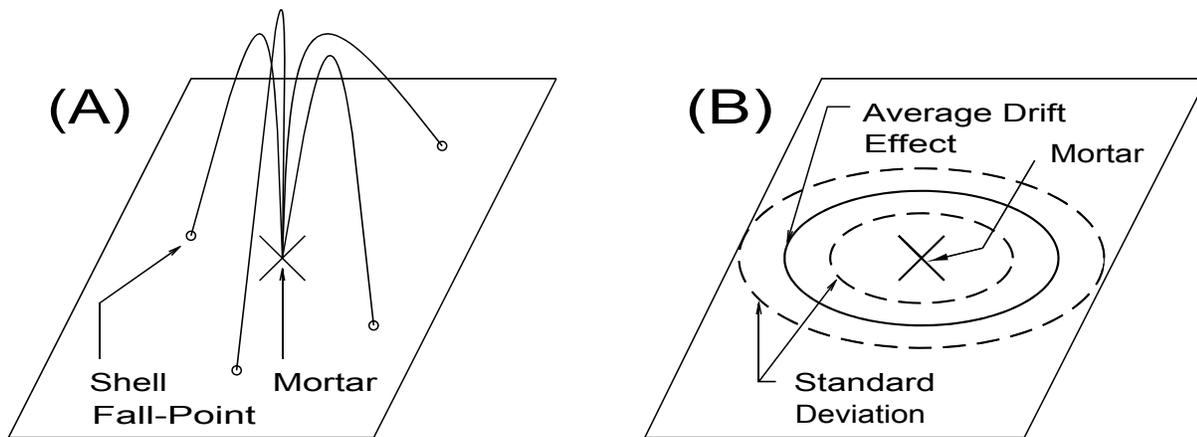


Figure 2. Method of determining aerial shell drift distance.

specifically made for these tests and were inert. They ranged in size from 7.6 cm (3 in.) to 15.2 cm (6 in.). These shells were also fired electrically.

Each test consisted of the firing of from 8 to 10 shells of one size. After firing and upon the shell's return to ground, the approximate point of impact of each shell was noted. Following the completion of firing of a series of shells of one size, the exact points of impact were determined relative to a coordinate grid system. This process is illustrated in Figure 2(A).

Data Reduction

If one could be assured that experimental conditions were ideal (perfect mortar alignment and absolutely no wind from the surface through the maximum height reached by the shell), there would be little data processing to perform. For each shell size, it would only be necessary to calculate the average displacements from the mortar, their standard deviation and standard error, as illustrated in Figure 2(B).

Unfortunately, conditions were not perfect; for example, there were usually winds aloft that pushed the shells somewhat off course. Accordingly, some mechanism was needed to separate systematic effects (such as winds aloft) from the randomly oriented shell drift effects. It was decided to shift the original coordinate grid in order to correct for systematic errors. This was accomplished by first calculating the mathematical center for the distribution of shell impact points and then assigning that as the origin of a new (shifted)

coordinate system. In effect, what that does is to say that the distribution of shell impact points in the shifted coordinate system is the distribution that would have occurred had there not been systematic errors.

If the tests for each type of aerial shell had included many shells, and if one could be certain that wind conditions did not change during the tests, then this grid adjustment method should work very well. However, in this study, only a limited number of shells were used and wind conditions probably did change at least a little during the time it took to fire the shells. Thus there is

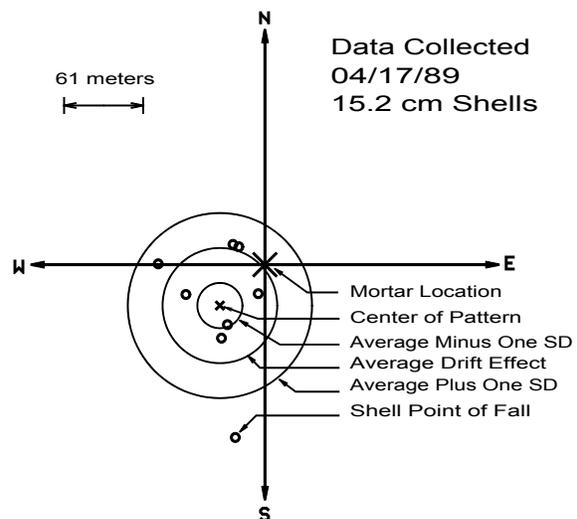


Figure 3. Illustration of 15.2-cm (6-in.) spherical aerial shell drift distance determination.

Table 2. 15.2-cm (6-in.) Spherical Shell Drift Effect Data.

Shell No.	Shell Displacement from Mortar (m)		Shell Displacement in Shifted Grid (m)		Distance from Center (m)
	North	East	North	East	
1	-22.9	-61.0	8.5	-26.2	27.4
2	0.6	-82.4	30.0	-47.6	57.3
3	15.6	-24.7	47.0	10.1	48.2
4 ^a	—	—	—	—	—
5	-46.1	-29.0	-14.6	5.8	15.9
6	-56.4	-33.6	-25.0	1.2	25.0
7	13.7	-20.1	45.1	14.6	47.6
8	-132.7	-22.9	-101.3	11.9	101.9
9	-22.3	-5.2	9.2	29.6	31.1
Average	-31.4	-34.8	≈0	≈0	44.2

(a) Shell burst at altitude, thus no drift data was produced.

some uncertainty as to how accurately the impact points in the shifted coordinate system represent actual shell drift effects. Having given this matter considerable thought, the authors feel that the average drifts presented in this paper probably are slightly under-estimated, while the reported standard deviations probably are slightly over-estimated. (For a more complete discussion of this subject, see Notes A and B of Reference 4.)

Figure 3 and Table 2 illustrate the process for the first set of shells fired, 15.2-cm (6-in.) spherical shells. The mortar was placed at the origin of the coordinate system, indicated by the large × in Figure 3. The points of fall of the shells fired are indicated as small circles, which are located primarily in the southwest quadrant. The center of the pattern of these points, the average displacements, was determined to be 31.4 m (103 ft) south and 34.7 m (114 ft) west and is indicated as the smaller × in Figure 3. Next, for each point of fall, the distance from the center of the pattern was determined and the average of those distances was calculated. In the case of these 15.2-cm (6-in.) spherical shells, the average is 44.2 m (145 ft) and is shown as the heavy solid line circle in Figure 3. [Note that in this initial test series an error had been made in the vertical positioning of the mortar. That is the primary reason for the center of the fall points being located about 47 m (154 ft) from the mortar.]

Even though the distribution of points about the average cannot be a true normal distribution, it is still useful to estimate the width of the distribu-

tion by calculating its standard deviation. The standard deviation, using the n-1 method, for the 15.2-cm (6-in.) spherical shells is 27 m (88 ft) and is shown in Figure 3 as the dashed circles. It is also useful to estimate the uncertainty in the average drift by calculating its standard error, which is 9.4 m (31 ft). Thus, the result for this series of shells is a drift effect of 44.2±9.4 m (145±31 ft).

Using this method, data was collected for nearly 50 groups of about 10 shells each.

Results

Spherical Shells, Average Drift Distance

Eight groups of spherical fireworks shells (75 shells in total) were fired during the determination of average drift distances. Table 3 and Figure 4

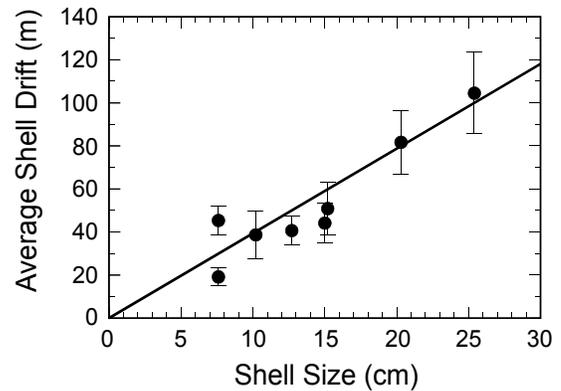


Figure 4. Average spherical aerial shell drift distance as a function of shell size.

Table 3. Average Spherical Aerial Shell Drift Distance.

Shell Size		Mean Drift		Standard Error ^(a)	
cm	(in.)	m	(ft)	m	(ft)
7.6	(3)	45.4	(149)	6.7	(22)
7.6 ^(b)	(3)	19.2	(63)	4.3	(14)
10.2	(4)	38.7	(127)	11.0	(36)
12.7	(5)	40.7	(140)	6.7	(22)
15.2	(6)	44.2	(145)	9.4	(31)
15.2 ^(c)	(6)	50.9	(167)	12.2	(40)
20.3	(8)	81.7	(268)	14.9	(49)
25.4	(10)	104.5	(343)	18.9	(62)

- (a) Standard error is equal to the observed standard deviation, using the n-1 method, divided by the square root of the number of shells fired for that measurement.
- (b) Because the first set of 7.6-cm (3-in.) aerial shells demonstrated unexpectedly high drifts, a second series of shells were fired the next day.
- (c) One set of 15.2-cm (6-in.) aerial shells was intentionally fired with a mortar tilt of 24°, causing the shells to be propelled down range.

present the results for these measurements of drift distance for spherical aerial shells. In Figure 4, the fit to the data was accomplished by a linear least squares regression, and equals 3.8 m of drift per cm (32 ft per in.) of shell size.

Statistical Distribution of Spherical Shell Drift Distances

Not enough aerial shells of any one size were fired to determine the nature of the statistical distribution for any individual size group. However, if it is assumed that the distributions are independent of shell size, the data from all shells fired could be used by standardizing the results for each of the groups. (This is a reasonable assumption, but not one that is assured.) To accomplish this standardization, each individual shell drift was expressed as a percent of the mean (average) for that size shell, as determined previously by a linear least squares fit of all the data. The standardized drift results for all 75 shells were then divided into twelve class intervals as shown in Table 4.

Figure 5 is a graph of the cumulative frequency of the standardized drift distances, against upper class interval limits, on a linear probability graph. The appearance of such a graph gives an indication of the nature of the statistical distribution. For example, a statistically normal distribution would appear as a straight line. Other distributions, such as log-normal, appear as curves unless plotted using a log axis. When a distribution

is normal bi-modal, it appears as two straight-line segments with different slopes. The distribution of aerial shell drift distances in Figure 5 has the appearance of two normal distributions, with the break occurring at the average shell drift distance. Thus from Figure 5, it can be seen that nearly 65% of the time shell drifts will be less than the mean, and that shell drifts as great as about 300% of the mean will occur nearly 1% of the time. In Table 4, it can be seen that the shells with the greatest relative drifts were for 7.6-cm (3-in.) and 10.2-cm (4-in.) shells. Thus it is possible that only the smaller sized shells experience such extreme drifts. Unfortunately, the present collection of data is not sufficiently large to allow that to be established with certainty.

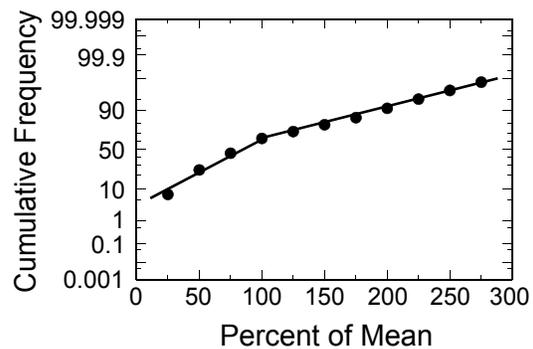


Figure 5. Cumulative frequency as a function of standardized spherical shell drift.

Table 4. Standardized Spherical Shell Drift Data by Class Interval.

Shell Size cm (in.)		Class Intervals (Percent of Mean Drift for Each Shell Size)											
		0– 25	25– 50	50– 75	75– 100	100– 125	125– 150	150– 175	175– 200	200– 225	225– 250	250– 275	275– 300
7.6	(3)	3	2	0	2	2	1	0	0	0	0	0	0
7.6	(3)	0	0	1	3	1	0	0	2	1	1	1	0
10.2	(4)	0	5	1	0	1	1	0	0	1	0	0	1
12.7	(5)	1	1	1	3	2	1	1	0	0	0	0	0
15.2	(6)	0	3	2	2	0	0	1	0	0	0	0	0
16.2	(6)	1	1	3	1	0	0	1	1	0	0	0	0
20.3	(8)	0	1	4	2	0	0	1	1	1	1	0	0
25.4	(10)	0	1	3	1	0	2	1	0	1	0	0	0
Totals		5	14	15	14	6	5	5	4	3	2	1	1
Cum. Tot.		5	19	34	48	54	59	64	68	71	73	74	75
Cum. %		7	25	45	64	72	79	85	91	95	97	99	100

10.2 cm (4 in.) Cylindrical Shells, Average Drift Distances

With cylindrical shells, length as well as diameter plays a role in determining drift distances. Also, shell mass and lift powder amounts can vary significantly with shell length. For this reason, the study of cylindrical shells was begun with a rather lengthy examination of the effects of shell length, shell mass, and lift amount for 10.2-cm (4-in.) shells. All of the shells used in this effort were plastic (so-called RAP™ Shells) with relatively smooth exterior surfaces, 9.2 cm (3.62 in.) in diameter, without lift-cup, and using 2F-A blasting Black Powder. Table 5 is a listing of the shell parameter values and the average drift distance for each group of ten shells fired.

In order to determine the functional relationship between drift distance and the various shell parameters, multivariate analysis was performed.⁹ Although 33 sets of 10 shells were fired in this test series, this is not a particularly large number considering the variability in the results and number of shell variables. Thus, it would be preferable to seek only linear relationships between drift distance and the shell variables. However, only for moderate amounts of lift powder is the relationship essentially linear; at low amounts of lift (2% or 3% of shell mass), drift distance falls very rapidly to near zero. In order to incorporate this effect, an additional (composite) variable, incorporating both shell lift and mass, was introduced into the multivariate analysis. The regression formula fitted in the analysis was

$$D_d = a + b \cdot M_s + c \cdot L_s + d \cdot M_l + e / (M_l - 0.03 \cdot M_s)$$

where,

- D_d = drift distance in m (ft),
- M_s = shell mass in gm (oz),
- L_s = shell length in cm (in.),
- M_l = lift amount in gm (oz), and

a,b,c,d and e are constants.

The constants, as determined by multivariate analysis, are:

- a = 16.4 m (54 ft)
- b = -0.0075 m/gm (-0.69 ft/oz)
- c = -0.36 m/cm (-3.0 ft/in.)
- d = 0.21 m/gm (18 ft/oz)
- e = -99 m/gm (9100 ft/oz).

The correlation coefficient for the multivariate regression is 0.86, which indicates quite a good fit of the data to Equation 1. (Note that a perfect fit would have produced a correlation coefficient of 1.00.) As an indication of the uncertainty in drift distance predictions made using Equation 1, it should be noted that the average deviation between the experimental results and predicted value was 22 percent.

If it is assumed that a typical 10.2 cm (4 in.) cylindrical shell weighs 454 g (16 oz), is 8.9 cm (3.5 in.) long, and uses 54 g (1.9 oz) of lift powder, Equation 1 predicts the average drift distance to be 18.7 m (61.3 ft). As shell weight, length and lift amount are varied, the average drift distance

Table 5. Shell Parameter Values and Drift Distance for 10.2-cm (4-in.) Cylindrical Shells.

Group Number	Shell Parameters							
	Shell mass		Shell Length		Lift Mass		Drift Distance	
	g	(oz)	cm	(in.)	g	(oz)	m	(ft)
1	250	(8.8)	7.6	(3.0)	25	(0.9)	10.6	(34.7)
2	250	(8.8)	7.6	(3.0)	38	(1.3)	20.2	(66.1)
3	250	(8.8)	7.6	(3.0)	50	(1.8)	27.5	(90.1)
4	250	(8.8)	7.6	(3.0)	75	(2.6)	35.2	(115.)
5	250	(8.8)	7.6	(3.0)	100	(3.5)	37.6	(123.)
6	250	(8.8)	15.2	(6.0)	38	(1.3)	7.8	(25.5)
7	250	(8.8)	15.2	(6.0)	75	(2.6)	13.6	(44.7)
8	250	(8.8)	30.5	(12.0)	38	(1.3)	9.1	(29.9)
9	250	(8.8)	30.5	(12.0)	75	(2.6)	13.0	(43.0)
10	500	(17.6)	7.6	(3.0)	25	(0.9)	3.8	(12.6)
11	500	(17.6)	7.6	(3.0)	38	(1.3)	13.3	(43.6)
12	500	(17.6)	7.6	(3.0)	50	(1.8)	11.2	(36.8)
13	500	(17.6)	7.6	(3.0)	50	(1.8)	19.5	(63.9)
14	500	(17.6)	7.6	(3.0)	50	(1.8)	17.9	(58.8)
15	500	(17.6)	7.6	(3.0)	50	(1.8)	21.4	(70.0)
16	500	(17.6)	7.6	(3.0)	50	(1.8)	17.2	(56.4)
17	500	(17.6)	7.6	(3.0)	75	(2.6)	25.4	(83.4)
18	500	(17.6)	7.6	(3.0)	100	(3.5)	32.1	(105.)
19	475	(16.8)	15.2	(6.0)	38	(1.3)	8.9	(29.4)
20	475	(16.8)	15.2	(6.0)	50	(1.8)	14.0	(46.0)
21	475	(16.8)	15.2	(6.0)	75	(2.6)	20.5	(67.4)
22	475	(16.8)	15.2	(6.0)	75	(2.6)	13.6	(44.6)
23	475	(16.8)	15.2	(6.0)	100	(3.5)	27.6	(90.5)
24	500	(17.6)	30.5	(12.0)	38	(1.3)	9.5	(31.3)
25	750	(26.5)	22.8	(9.0)	38	(1.3)	4.7	(15.4)
26	750	(26.5)	22.8	(9.0)	75	(2.6)	14.5	(47.6)
27	925	(32.6)	7.6	(3.0)	50	(1.8)	11.4	(37.4)
28	925	(32.6)	7.6	(3.0)	100	(3.5)	18.6	(61.0)
29	1000	(35.2)	15.2	(6.0)	50	(1.8)	9.0	(29.5)
30	1000	(35.2)	15.2	(6.0)	100	(3.5)	24.8	(81.6)
31	980	(34.6)	30.5	(12.0)	50	(1.8)	7.2	(23.6)
32	980	(34.6)	30.5	(12.0)	75	(2.6)	15.4	(50.5)
33	980	(34.6)	30.5	(12.0)	100	(3.5)	20.1	(66.0)

should change as suggested by the constants b through e above. Figures 6A through 6C demonstrate the expected result of varying these shell parameters.

Drift Distance Reproducibility

In a brief discussion of aerial shell drift experiments, Shimizu reports¹² that significantly different test results were observed on different occasions. Specifically, he observed that the drift of dud shells was about twice as great on one occasion as it was on another. He speculated that a possible reason for this might have been turbulent

air currents experienced on one of the days. The authors also observed a similar situation; a second set of measurements was made on 7.6-cm (3-in.) spherical shells because results from the first set were unexpectedly high. The two sets of drift distances differed by more than a factor of two. This was enough greater than the calculated standard errors (see Figure 4), to suggest that the difference may not be the result of a random statistical occurrence. A brief attempt was made to look at this further. Five identical groups of shells, numbers 12 through 16 in Table 5, were fired on five different days. Figure 7 displays the average drift

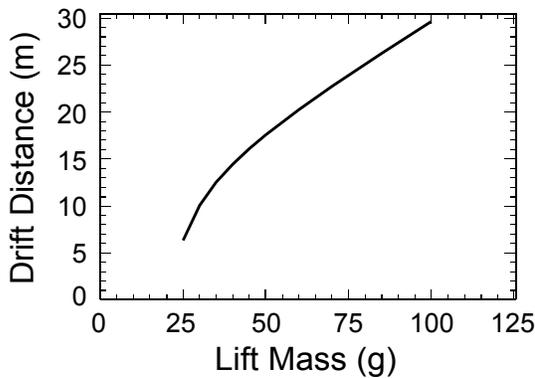


Figure 6A. Effect of varying lift powder mass for a typical 10.2 cm (4-in.) cylindrical shell.

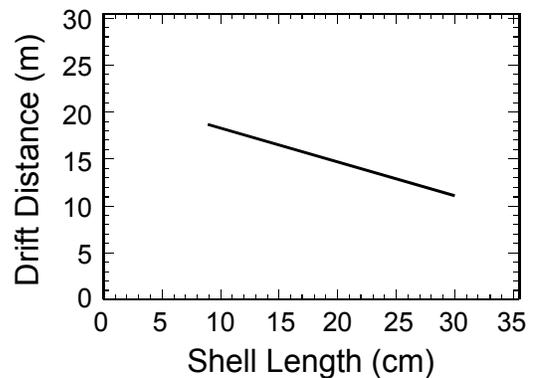


Figure 6B. Effect of varying shell length for a typical 10.2 cm (4-in.) cylindrical shell.

distances for these five measurements, along with their standard errors. Again there is about a factor of two in the spread of the data. However, considering the standard errors, one cannot be certain that the difference was more than a random statistical occurrence.

Cylindrical Shells, Average Drift Distance

In addition to the drift distance for 10.2-cm (4-in.) cylindrical shells, data was also collected for 7.6, 12.7, and 15.2-cm (3, 5, and 6-in.) shells. These too were inert shells made especially for testing. Varying shell parameters significantly affects drift distance, thus information on shell diameter, shell length, shell mass and mass of 2 F-A lift powder was included in Table 6 along with the results of drift distance measurements.

Table 6 reports results for a series of 15.2-cm (6-in.) shell tests, using varying amounts of lift

powder. The observed drift distances follow a relationship similar to that shown in Figure 6A. If it is assumed that the typical amount of lift used for 15.2-cm (6-in.) cylindrical shells, with other parameters as listed in Table 6, is 112 g (4 oz) then the expected drift distances will be about 43 m (140 ft). This result and the others reported in Table 6 are plotted in Figure 8, and have the appearance of a roughly linear relationship with a slope of approximately 2.4 m per cm (20 ft per in.) of shell size.

Comparison with the Results of Others

Approximately 30 years ago T. Shimizu, working at the request of Professor S. Yamamoto at Tokyo University,^{6,7} studied the drift of spherical aerial shells for the purpose of determining appropriate separation distances. In order to simulate typical conditions, many of the measurements

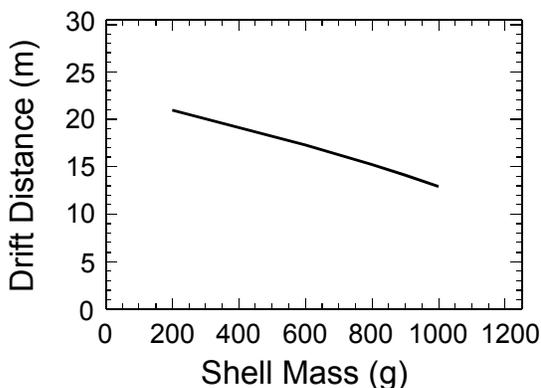


Figure 6C. Effect of varying shell mass for a typical 10.2-cm (4-in.) cylindrical shell.

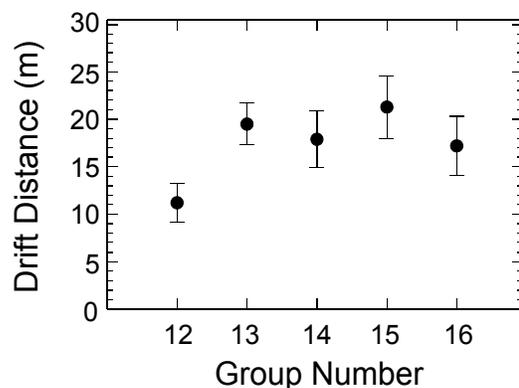


Figure 7. Reproducibility of results from five groups of ten identical 10.2-cm (4-in.) cylindrical shells.

Table 6. Shell Parameter Values and Drift Distances for Cylindrical Shells.

Shell Size		Shell Diameter		Shell Length		Shell Mass		Lift Mass		Drift Distance	
cm	(in.)	cm	(in.)	cm	(in.)	g	(oz)	g	(oz)	m	(ft)
7.6	(3)	6.7	(2.62)	6.9	(2.7)	180	(6.5)	28	(1)	20	(64)
7.6	(3)	6.7	(2.62) ^[a]	6.9	(2.7)	180	(6.5)	28	(1)	5	(49)
10.2	(4)	9.2	(3.62) ^[b]	8.9	(3.5)	460	(16.)	56	(2)	190	(63)
12.7	(5)	11.4	(4.5)	10.2	(4.0)	910	(32.)	84	(3)	30	(98)
12.7	(5)	11.4	(4.5) ^[a]	10.2	(4.0)	910	(32.)	84	(3)	36	(120)
15.2	(6)	14.1	(5.56)	12.7	(5.0)	1800	(64.)	75	(2.7)	22	(72)
15.2	(6)	14.1	(5.56)	12.7	(5.0)	1800	(64.)	100	(3.6)	36	(120)
15.2	(6)	14.1	(5.56)	12.7	(5.0)	1800	(64.)	130	(4.5)	58	(190)
15.2	(6)	14.1	(5.56)	12.7	(5.0)	1800	(64.)	130	(4.5)	49	(160)

[a] These shells were fired from non-vertical (tilted) mortars.

[b] Typical shell parameters and calculated drift distances using Equation 1.

were made using mortars angled to about 10° and in many cases a significant wind was blowing. In some tests, efforts were made to restrict the normal spin of the shells after firing; also the grid adjustment method used in this study was not employed. Thus although they were excellent studies, most of the data is not directly comparable with the results of this study. However, Table 7 presents the results from those cases that are the most comparable.

On average, the drift distance results of Yamamoto (Shimizu) are about 5 percent greater than those predicted from this study. Considering the differences between the two bodies of work, this is amazingly good agreement and serves to

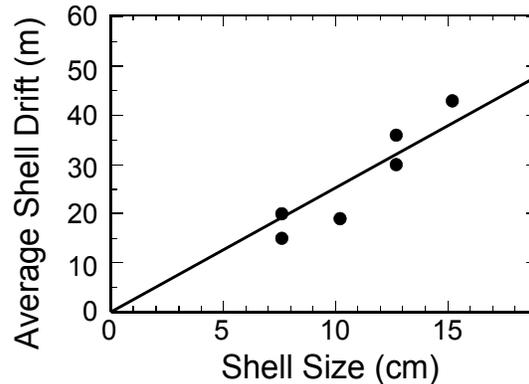


Figure 8. Average cylindrical aerial shell drift distance as a function of size.

Table 7. Comparison of the Yamamoto Spherical Shell Drift Results with Those of This Study.

Conditions	Drift Distance		This Study's Results		Difference
	m	(ft)	m	(ft)	
5 Light Shells with smoke candle attached (wind = 0.7 m/s) (size = 3 in.)	25.0	(82)	29.3	(96)	-15%
5 Heavy Shells with smoke candle attached (wind = 0.7 m/s) (size = 3 in.)	38.9	(128)	29.3	(96)	+33%
15 Heavy Shells with smoke candle attached (wind = 0.7 m/s) (size = 3 sun)	35.8	(117)	35.1	(115)	+2%
15 Light Shells with smoke candle attached (wind = 2 m/s) (size = 3 sun)	39.9	(130)	35.1	(115)	+13%
15 Heavy Shells with smoke candle attached (wind = 2.5 m/s) (size = 5 sun)	54.2	(178)	58.5	(192)	-7%

(a) Note that 1 sun = 3 cm (1.2 in.).

increase the authors' confidence in their results.

In 1989, E. Contestabile, at the Canadian Explosives Research Laboratory, conducted a series of aerial shell ballistics tests.⁸ Only 15.5-cm (6-in.) shells were used. They weighed 1340 g (48 oz) and were fired from a 4-m (13-ft) long mortar using 42.5, 56.7 or 99.1 g (1.5, 2.0, or 3.5 oz) of lift powder. The shells were of an unusual geometry, having a relatively short cylindrical wall and domed ends, one end having a concave recess to contain the lift charge. Thus, it would be anticipated that the drift distances for these intermediately shaped shells might be somewhere between those reported here for spherical and cylindrical shells. The results from the Contestabile tests are listed in Table 8.

Table 8. Contestabile Shell Drift Results.

Lift Weight g (oz)	Number of Shells Fired	Drift Distance m (ft)
42.5 (1.5)	7	11.8 (39)
56.7 (2.0)	7	28.2 (93)
99.2 (3.5)	12	53.5 (176)

In order to compare Contestabile's data with the results from this study, it is necessary to adjust for what would be expected for a more typical 15.2-cm (6-in.) cylindrical shell. In the present study this was considered to be a shell weighing about 1800 g (64 oz) and using 112 g (4.0 oz) of lift powder. Based on the effect of shell mass and lift observed for 10.2-cm (4-in.) shells, and the effect of lift mass for 15.2-cm (6-in.) shells observed in this study, the authors estimate that Contestabile would have observed a drift distance of about 46 m (150 ft) for such shell and lift mass. This compares well with the prediction from the present study of 58 m (190 ft) for spherical shells and 36 m (120 ft) for cylindrical shells, particularly when it is recalled that the Contestabile shells

were expected to experience drifts somewhere between those for cylindrical and spherical shells.

Effect of Mortar Tilt on Drift Distance

On three occasions, groups of the same size shells were fired from both vertical and tilted mortars. This brief study was conducted to discover the approximate magnitude of any strong dependence of shell drift distance on mortar tilt angle. Table 9 lists the results of this study. Although some differences were observed, the results are not consistent. Thus it would seem that if there is a dependency of drift distance on mortar tilt angle, the effect is too small to have been observed in this brief study.

Discussion

In research there always seems to be more data that could (should) be collected. However, the data collected to date are probably sufficient and should be used to consider the important question of the adequacy of spectator separation distances. Unfortunately, doing that is more complex than might at first be realized. For example, it involves making assumptions about such things as:

- How accurately can a typically skilled display operator predict the ideal trajectory of aerial shells?
- How precisely can a typically skilled display operator align his mortars?
- To what extent will mortar alignment change during firing?
- How different are winds aloft likely to be than those experienced at ground level, which were considered in deciding how the mortars should be aimed?
- What percentage of dud shells falling outside the secured boundary is acceptable, recognizing that choosing 0% would probably require

Table 9. Effect of Mortar Tilt on Drift Distance.

Shell cm (in.)	Shell Shape	Mortar Tilt Angle (°)	Drift Distance		Difference
			m	(ft)	
7.6 (3)	Cyl.	0	19	(64)	- 23%
		15	15	(49)	
12.5 (5)	Cyl.	0	30	(98)	+ 21%
		10	36	(119)	
15.2 (6)	Sph.	0	44	(146)	+ 15%
		24	30	(167)	

about 18 m per cm (150 ft per in.) of shell size?

Because of these complexities, a discussion of appropriate spectator separation distances is beyond the scope of this article. Hopefully, the authors or others will soon undertake this important task.

Another area of application for the results reported here is in statistically predicting the trajectory of aerial shells. Armed with information about drift distances, it is possible to use a relatively simple computer model to predict the average trajectory of aerial shells and then superimpose on those results the empirically determined and statistically distributed drift distances.³ This has been performed in a number of cases to determine the likelihood of various accident scenarios.

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