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Typical Mortar Recoil Forces for Spherical Aerial Shell Firings[a]

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(Included in the text of this article are a series of notes. These notes present ancillary information that may be of interest to some readers but are not strictly needed within the context of this article. Thus readers should feel free to ignore the notes unless they desire more information.)

One of the more common requests for information regards the recoil force produced when aerial shells are fired from mortars. Generally the concern is whether some support structure (e.g., roof top, platform or barge deck) will safely accommodate the dynamic load produced as shells of various sizes are fired from mortars placed upon the support structure. Providing a precise answer can be a complex engineering problem, requiring information that is not readily available. However, providing reasonable estimates for the recoil forces produced by the firing of typically performing aerial shells is a relatively easy matter. This article provides those approximate values for typical 3- through 12-inch (75- through 300-mm) spherical aerial shell firings. (These values are only for single break spherical shells; they are not for cylindrical shells or for so-called stacked, double-bubble, or peanut spherical shells.)

Pressure is defined as the force applied per unit area, thus in the English system it has units such as pounds (force) per square inch (area). Accordingly, to calculate the total force (F) produced by a pressure acting on some surface, simply multiply the pressure being applied (P) by the area (A) over which it is acting, i.e.

$$F = P \times A \qquad (1)$$

In the case of an aerial shell firing, if P is the pressure developed inside the mortar[b] and A is the inside cross-sectional area of the mortar, a reasonable estimate of recoil force can be calculated using equation 1.[c] There is no net contribution to recoil force produced by the pressure acting radially on the inside walls of the mortar. This is because the force against each small portion of mortar wall is balanced by an equal but opposite force on the portion of mortar wall directly across from it.[d] Accordingly, all that is needed to produce the estimates of recoil force is knowledge of the pressure profile in the mortar (i.e., internal mortar pressure as a function of time) and the inside cross-sectional area of the mortar (calculated using the internal diameter of the mortar).

Figure 1 illustrates internal mortar pressure as a function of time during the firing of a typical 4-inch (100-mm) aerial shell. First there is an extended period of time (t_0 to t_i) during which there is effectively no pressure rise inside the mortar. This length of time is commonly 0.01 to 0.02 second. This corresponds to the time taken for fire to spread among and ignite the grains of lift powder, before there is sufficient gas production to cause a detectable pressure rise in the mortar. After that, there is a rapidly accelerating increase in mortar pressure up to some peak value, generally occurring over a period of 0.005 to 0.01 second (t_i to t_p).[e] Next, as the shell accelerates upward (increasing the volume below the aerial shell) and the rate of gas production begins to lag, mortar pressure typically drops during the

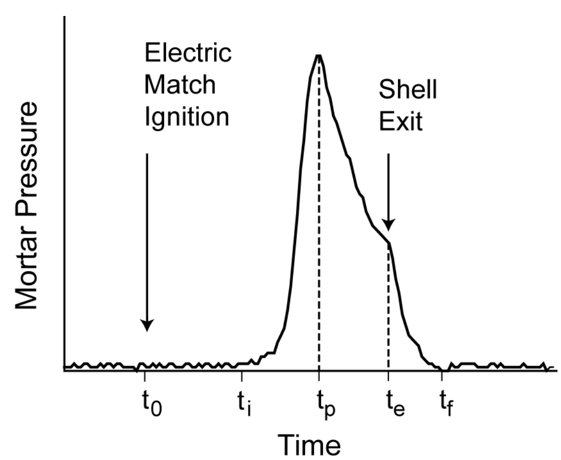

Figure 1. A somewhat typical internal mortar pressure profile during the firing of an aerial shell.

time before the shell exits the mortar (t_p to t_e). This pressure drop generally continues for 0.005 to 0.01 second.[f] Finally, the exiting of the shell causes an even more precipitous drop in mortar pressure back to ambient levels over a time period of 0.005 to 0.01 second (t_e to t_f). The total duration of the pressure pulse (t_i to t_f) typically ranges from 0.02 to 0.03 second.

While the pressure developed in mortars during a shell firing does depend on shell size, both the general shape of the pressure profile and its duration are mostly independent of shell size.[g] Peak mortar pressures were measured during the firing of 136 spherical aerial shells (ranging in size from 3 to 12 inches [75 to 300 mm]) and were found to be well fitted by a simple linear relationship with shell size.[1] Specifically, the peak pressure was found to increase by approximately 13.6 psi per shell inch (94 kPa/25 mm), as is shown in column 2 of Table 1. Using equation 1, these peak pressures can be converted to peak recoil forces, simply by multiplying peak pressure by the inside cross-sectional area of the mortar, see columns 3 and 4 of Table 1.

For most support structures (e.g., a reasonably substantial building roof) the response of the structure to the recoil force of a mortar firing will not be proportional to the peak force but rather to the impulse delivered.[i] (The impulse delivered is equal to the product of the average force exerted, times the total duration of the force.[j]) In testing it has been found that the average recoil force is approximately 45% of the peak force, mostly independent of shell size. Accordingly, the average force reported in column 5 of Table 1 is just 45% of the peak force value for that size shell in column 4. It has also been found that the typical duration of the recoil force is approximately 0.025 second.[k] Thus the recoil impulse for each size of typical spherical shell (column 6 of Table 1) is the average pressure value of column 5 multiplied by 0.025 second.

Note that the mortar recoil impulse values

Table 1. Mortar Pressure, Recoil Force and Impulse for Various Size Aerial Shell Firings.

Mortar Size [ID]		Peak Pressure (i)	Mortar Area (ii)	Peak Force (iii)	Average Force (iv)	Impulse (v)	Equivalent 4 foot drop (vi)
(in.)	(mm)	(psi)	(in.^{2})	(lbf)	(lbf)	(lbf s)	(lbf)
3.0	75	41	7.1	290	130	3.3	7
4.0	100	54	13.	700	320	7.9	16
5.0	125	68	20.	1,400	630	16	32
6.0	150	82	28.	2,300	1000	26	52
8.0	200	106	50.	5,300	2400	60	120
10.0	250	136	79.	11,000	5000	120	240
12.0	300	163	110.	18,000	8000	200	400

i) From reference 1, to the nearest 1 psi. Note to convert from psi to kPa, multiply by 6.9.

ii) Mortar Area, $A = \pi \times d^2/4$, where d = internal mortar diameter. Note to convert from in.^{2} to cm^2, multiply by 6.4.

iii) Peak Force, F = (Peak Pressure) × (Mortar Area) = $P \times A$. The values of peak force are reported to only two significant figures. Note to convert from lbf (pound force[h]) to N, multiply by 4.45.

iv) In examining a number of mortar pressure profiles, the average force exerted over the duration of the mortar's recoil was typically found to be approximately 45% of the peak force produced. The value for each average force in Table 1 is equal to 45% of the Peak Force for that size shell and is reported to only two significant figures. Note to convert from lbf to N, multiply by 4.45.

v) Impulse = (Average force) × (Typical recoil duration), where the typical duration was found to be approximately 0.025 second. The values of impulse are reported to only two significant figures. Note to convert from lbf·s to N·s, multiply by 4.45.

vi) This is the weight of a solid object that, if dropped from a height of 4-feet (1.2 m) on to the support structure, will deliver the same impulse as the firing of that size of a typical single-break spherical aerial shell. This weight in pounds is numerically equal to twice the impulse given in column 6 of Table 1. Values are given to two significant figures, but rounded to next largest pound. (See note k for an explanation.) Note to convert from lbf to kg, multiply by 2.2.

(column 6 of Table 1) approximately double for each increase in shell size. This means that the stress delivered to a support structure also more or less doubles with each incremental increase in shell size. That is to say, the firing of a 5-inch (125-mm) spherical shell from a mortar will typically deliver about twice the stress (impulse) as firing a 4-inch (100-mm) shell. While this is useful information, it says nothing about whether the support structure is sufficiently strong to safely withstand the intended size of shell firings. To make that determination, a structural engineer will need to be consulted to consider data such as in Table 1. However, to provide points of comparison with which most readers will be familiar, column 7 has been included in the table.[l] These are solid weights that, when dropped from a height of four feet (1.2 m), will deliver approximately the same impulse as will typically be produced by the firing of that size single-break spherical aerial shell.

For example, from the last column of Table 1, if the support structure can safely withstand the drop of a fairly rigid[m] 7-pound (3.2-kg) mass from a height of four feet (1.2 m), then it can probably survive the firing of a typical 3-inch (75-mm) single-break spherical aerial shell. If you can do the same with a 400 pound (182 kg) rigid mass, then the structure can probably survive the firing of a typical 12-inch (300-mm) spherical aerial shell. However, it is important to consider that:

1) It is implicitly assumed that the size (diameter) of the mass that is dropped is approximately the same as that of the bottom of the mortar being considered.

2) The data in Table 1 is for the firing of *typical single-break spherical aerial shells*,[a] which means that roughly half of the shell firings must be expected to produce forces greater than those presented in Table 1. And some properly functioning single-break spherical shells will produce twice the average recoil force.

3) The values in Table 1 do not take into consideration the possibility of aerial shell malfunctions, which could produce forces substantially in excess of those listed.

4) It is a standard engineering practice to design-in a safety margin of at least two or three.

Although this article provides some approximate guidance regarding mortar recoil forces[n] and the needed strength of support structures for the safe firing of single-break spherical aerial shells,[o] note that neither author is a mechanical or structural engineer. Thus a mechanical or structural engineer needs to be consulted prior to relying on the information presented in this article.

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Notes

a) Recent measurements of the mortar pressures produced upon firing spherical aerial shells suggest that some manufacturers are using significantly more lift powder. The result is that the previously reported typical mortar pressures[1] may presently under estimate the pressures now typically being produced. If this is now the case, then the "typical mortar recoil forces" being reported in this article are also under estimates.

b) In this case it is gauge pressure (i.e., the pressure over and above atmospheric pressure), and not absolute pressure, that is used to calculate the resulting force.

c) At least two other factors act to slightly modify the recoil force predicted by equation 1. First, upon firing an aerial shell, there is an upward rush of gas escaping through the gap between the shell's casing and the mortar wall. Some of the motion of this gas flow is communicated to the wall of the mortar. The result of this first factor is that a small upward force is produced that slightly counteracts the much more substantial downward recoil force being produced. The second factor is a result of the constriction provided by the aerial shell that acts somewhat as a plug nozzle in a rocket motor, such that some added thrust may be produced by the escaping lift gas (i.e., the thrust coefficient may be slightly greater than 1.0). The result of this second factor is that a little greater downward force may be produced. For the purpose of this article, both of these minor effects are ignored.

d) The net result of the balanced (equal and opposite) forces on the mortar wall is to produce a tensive force (called hoop stress) in the wall of the mortar. This is only a concern when that force is greater than the tensile strength of the mortar, in which case the mortar will burst. For information on calculating burst strength of pipes (i.e., mortars), see any standard engineering text or any edition of the *Machinery's Handbook* (published by Industrial Press, Inc.) under "strength of materials".

e) Some have attributed this effect to so called "choked flow", apparently thinking that once the velocity of escaping gas reaches the speed of sound and no longer increases, the gas flow ceases to increase even as the pressure continues to increase. There is a limit to gas flow *velocity* at the point of constriction posed by the aerial shell, when that flow velocity reaches the speed of sound under the conditions in the mortar. Nonetheless, because the *density* of the gas continues to increase with pressure, the *mass flow rate* continues to increase even though the velocity of the flow does not. (For more information about choked flow, see reference 3.)

f) With some types and granulations of lift powder, the rate of rise in mortar pressure will be less, and the aerial shell may exit the mortar while the pressure is still increasing. In that case there will be no time interval between the peak recorded pressure and the exit of the shell (i.e., the time from t_p to t_e can be zero).

g) The shape, magnitude and duration of the pressure profile depends on factors such as the characteristics of the lift charge (e.g., its granulation and to a lesser extent its mass), temperature, dead volume under the shell (also called loading space), the mass of the shell, and the size of the gap between the shell and mortar wall.

h) In the Imperial (English) System of units, the pound unit can be either a unit of force or a unit of mass. To help avoid confusion, the abbreviation for pound force is lbf.

i) When a force is applied to a structure, the response of the structure depends on whether the duration of the applied force is greater than or less than the resonant period of the structure.[2] If the duration of the applied force is long in comparison to the resonant period, the response is proportional to the peak force. If the duration of the applied force is less than the resonant period, the response of the structure is proportional to the impulse delivered. Over the course of many measurements of pressure profiles during the firing of aerial shells,[1] it was found that the total duration of the pressure pulse in the mortar (i.e., the duration of the recoil force produced) averaged approximately 0.025 s, and the duration was found to be essentially independent of shell size. For most supporting structures, resonant periods are probably at least 10 times longer, especially for substantial (i.e., massive) structures. Thus the duration of mortar recoil events is much shorter than the resonant period of typical structures, and the structure's response to mortar recoil will be proportional to recoil impulse and not peak recoil force.

j) More correctly from a mathematical standpoint, when pressure is not constant, impulse is equal to the integral of ($P\,dt$).

k) In note g, it was mentioned that there are a number of factors that affect peak pressure and the duration of the pressure pulse. However, in terms of impulse, these factors are less important. This is because those things that tend to increase peak pressure also tend to decrease the duration of the pressure pulse, thus tending to cancel-out the overall effect on impulse.

l) The impulse (I) delivered in stopping a moving object is equal to its momentum (M) since its final velocity is zero. Momentum is equal to the mass (m) of the object times it velocity (v). Further, mass is equal to an object's weight (w) divided by the acceleration due to gravity (g), and the velocity of an object that is dropped is equal to the acceleration due to gravity times the time (t) the object is falling. Thus,

$$I = M = m \times v = \left(\frac{w}{g}\right)(g \times t) = w \times t \qquad (2)$$

The distance (h) an object will fall during a time interval, equals one half the acceleration due to gravity times the square of the time during which it fell. Thus:

$$h = \tfrac{1}{2} \times g \times t^2 \qquad (3)$$

Solving for *t*.

$$t = \left(2\left(\frac{h}{g}\right)\right)^{1/2} \quad (4)$$

For a weight dropped from a height of 4 feet (1.2 m), with the acceleration due to gravity of 32 feet per second squared (9.8 Nm/s^2), the time taken is ½ second.

$$t = \left(\frac{8}{32}\right)^{1/2} = \text{½ second} \quad (5)$$

Thus, substituting for time (*t*) back into equation 2, the momentum (and impulse) produced by a weight dropped from a height of 4 feet (1.2 m) is numerically equal to one half of that weight. Drops from any other height can be considered in a similar fashion. (For a more complete discussion consult any college level general physics text.)

m) The duration of the recoil force from firing an aerial shell is quite short (approximately 0.025 second, with the peak force coming early and lasting much less than the total duration). A fairly rigid mass is specified as the test object because it is thought that dropping a fairly rigid object will generate a force profile somewhat similar to that of a shell firing. To the contrary, a non-rigid mass such as a sand bag may deliver the same impulse as that of a rigid mass, but the duration of force produced will be longer for the sand bag as its contents shift upon impact. Accordingly, depending on the detailed nature of the supporting structure, the sand bag probably will not produce a stress on a support structure sufficiently similar to that of a shell firing.

n) Subsequent to writing this article, the authors found some published data on the recoil forces produced by large caliber spherical aerial shells. Those data were found to be in reasonable agreement with the estimates included in this article.[4]

o) Regarding the firing of single-break cylindrical shells some very limited testing suggests that the mortar pressures developed are roughly double that of spherical shells of the same size. Thus as a rough approximation, the values given in Table 1 would need to be doubled for the firing of typical single-break cylindrical shells.

References

1) K. L. and B. J. Kosanke, "Peak Mortar Pressures When Firing Spherical Aerial Shells", *Fireworks Business*, No. 197, 2000; also in *Selected Pyrotechnic Publications of K. L. and B. J. Kosanke, Part 5 (1998 through 2000)*, Journal of Pyrotechnics, 2002.

2) G. F. Kinney and K. J. Graham, *Explosive Shocks in Air*, 2nd ed., 1985, pp 187–189.

3) L. Weinman, "Choked Flow, A Frequently Misunderstood Term", *Journal of Pyrotechnics*, No. 19, 2004.

4) K. L. and B. J. Kosanke, "Confirmation of Mortar Recoil Information", *Fireworks Business*, No. 253, 2005.